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APPLICATION OF DERIVED PROBABILITY AND DYNAMIC PROGRAMMING TECHNIQUES TO PLANNING REGIONAL STORMWATER MANAGEMENT SYSTEMS

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ABSTRACT

There exists a need on the part of land developers or municipalities responsible for stormwater servicing master planning to reduce the costs associated with the initial construction of stormwater facilities as well as with their operation and maintenance. Common integral components of stormwater control systems include stormwater management ponds for water quality control. These ponds may occupy valuable urban land and, hence, it is desirable to minimize the land coverage of these facilities while simultaneously satisfying water quality control objectives. The employment of optimization techniques in the planning and design process can thus play an important role by reducing the costs associated with the implementation of such facilities.

This paper presents an optimization methodology for single catchments using a single stormwater quality control pond. This methodology is then further developed for a multiple parallel catchment (each with a single pond upstream of its outlet) optimization procedure employing principles of dynamic programming. The principal constraint of the problem formulation is to meet a specified pollution control level at the outfall to a receiving water body. The optimization technique employs analytical probabilistic models for stormwater management planning and analysis which are in a mathematically closed form and thus easily integrated into an optimization framework. The costs explicitly considered are land-associated costs and construction costs. Operation and maintenance costs can be incorporated into the framework if desired.

KEYWORDS

Detention ponds; optimization; pollution control; probabilistic models; regional planning; stormwater management; urban drainage; water quality.

INTRODUCTION

One of the most commonly used means of mitigating pollutional impacts from urban drainage is the implementation of stormwater quality control ponds. Although these ponds enhance the sustainable development of urban areas, to the developer the ponds are typically viewed as a loss of potentially developable land as well as an added cost due to the construction of the facility.

The objective function of the land developer is to minimize the cost associated with the construction of stormwater management ponds while satisfying environmental regulations. Typical costs of concern are the cost of the land occupied by the ponds and the cost of construction of the ponds. The cost of land can be taken as the purchase value of the land or the lost income resulting from the undevelopable land occupied by the ponds whereas the cost of construction of the pond can be related to the volume of the pond in terms of an excavation cost per unit volume. This paper presents a methodology for optimizing pond geometries (by minimizing the above-mentioned costs) for a single catchment as well as for a set of parallel catchments. The results from a case study which involves three parallel catchments are presented.

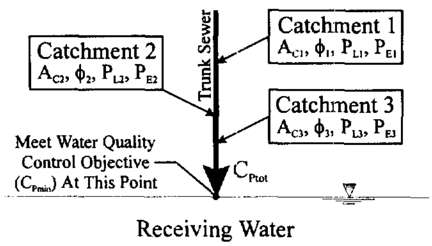

Figure 1. Schematic representation of problem.

OPTIMIZATION METHODOLOGY

Consider a system of parallel catchments, each with a stormwater quality control pond upstream of its outlet, draining into a common collection system (e.g., a trunk sewer) which subsequently conveys the stormwater into a receiving water. This scenario is illustrated in Figure 1 for a system of three catchments. Each catchment is defined spatially based on its local topography, its runoff coefficient (f), as well as its land and construction costs. Further consider that the discharge from the trunk sewer is required to meet or exceed a specified quality control objective (C_{Pmin}). Based on this scenario, it is conceivable that different levels of control can be provided at the outlet of each catchment such that the overall control meets or exceeds the environmental regulation. Moreover, the total cost of implementing the stormwater quality control ponds for all catchments may be minimized by an optimal blend of pond geometries in each of the catchments.

The water quality control constraint can be expressed mathematically as:

$$C_{Ptot} = \frac{\sum_{i=1}^{n} (C_{Pi} \cdot A_{Ci})}{\sum_{i=1}^{n} A_{Ci}} \geq C_{P\,min} \tag{1}$$

C_{Pmin} is the minimum allowable level of pollution control that is to be provided prior to discharge into the receiving water, C_{Ptot} is the area-weighted average level of pollution control provided by the catchments contributing to the trunk sewer, C_{Pi} and A_{Ci} are the level of pollution control provided and the surface area (ha) of the i^{th} catchment, respectively.

The objective of the optimization methodology is to minimize the total cost of implementing the stormwater management ponds in each of the catchments. The cost of a single pond can be estimated by

$$P_{Pond} = P_L \cdot \frac{A_C \cdot S_A \cdot 10}{h_A} + P_E \cdot A_C \cdot S_A \cdot 10 \tag{2}$$

where A_C and S_A are the catchment area (ha) and active storage volume of the pond (in mm uniformly distributed over the catchment area), respectively. The terms P_L ($\$/m^2$) and P_E ($\$/m^3$) are the land cost and the construction (excavation) cost, respectively. The decision variables of this design optimization procedure include the storage volume of the pond, S_A, the controlled release rate from the pond (W, in mm uniformly distributed over the catchment per hour) and the pond depth (h_A, in m).

Using Equation 2 coupled with the pollution control model presented in the next section, a least-cost combination of the decision variables (S_A, Ω and h_A) can be obtained for a single catchment pond based on the level of pollution control (C_P) required from the pond. This least-cost optimization is easily obtained through the implementation of numerical search methods since the governing equations of the model are closed-form expressions and thus computationally efficient. Furthermore, a least-cost combination of ponds in multiple catchments may be obtained through a dynamic programming optimization formulation of the problem as depicted in Figure 1. The objective of the overall optimization (i.e., multiple catchment) is then to minimize the total cost associated with ponds in each catchment which can be represented by

$$TC = \sum_{i=1}^{n} (P_{Pond})_i = \sum_{i=1}^{n} \left(P_{Li} \cdot \frac{A_{Ci} \cdot S_{Ai} \cdot 10}{h_{Ai}} + P_{Ei} \cdot A_{Ci} \cdot S_{Ai} \cdot 10 \right) \quad (3)$$

It should further be noted that it is assumed herein that there is no further cost associated with different rates of release from the pond since these quality control facilities are typically designed for long stormwater detention times where release rates are relatively low. It is therefore assumed that the downstream conveyance system and/or stormwater quantity control facility can readily handle the discharges from the quality control ponds presented herein.

MODEL FOR POLLUTION CONTROL PERFORMANCE

Analytical probabilistic models for urban stormwater management offer the advantage of estimating long-term quality and quantity control performance of urban drainage systems using a limited number of input parameters as well as having minimal computational requirements. This renders their usage ideal for planning level analyses where more costly and time consuming full-scale continuous simulations are typically avoided. These models, which incorporate continuous rainfall records to develop meteorological statistics, are expressed in a mathematically compact form and can thus be easily implemented into an optimization framework.

This paper focuses on the use of extended detention dry ponds for stormwater quality management. Adams (1996) presents the following expressions for predicting the total suspended solids removal from extended detention dry ponds:

$$C_P = E_d \left\{ 1 - \left[\frac{\frac{\lambda}{\Omega}}{\frac{\lambda}{\Omega} + \frac{\zeta}{\phi}} \right] \left[\frac{\frac{\psi}{\Omega} + \frac{\zeta}{\phi} \exp\left[-\left(\frac{\psi}{\Omega} + \frac{\zeta}{\phi} \right) S_A \right]}{\frac{\psi}{\Omega} + \frac{\zeta}{\phi}} \right] \right\} \quad (4)$$

where

$$E_d = \sum_i F_i \left\{ 1 - \left[1 + \frac{V_{si}}{n \cdot h_A} \cdot \frac{S_A}{2 \cdot \Omega} \right]^{-n} \right\} \quad (5)$$

where C_P is the pollution control expressed as a fraction, n is the turbulence factor expressing the settling characteristics of the pond, V_{si} and F_i are the average settling velocity and the fraction of total mass contained in the i^{th} size fraction, respectively, using a settling velocity distribution of pollutants in stormwater. The terms λ, ζ and ψ are parameters for the exponential distribution of probability density of the meteorological characteristics of rainfall event duration, volume and interevent time, respectively. For detailed explanations of the model expressions and parameters, the reader is referred to Adams (1996).

Papa *et al.* (1997) present a comparison of the above analytical model results with continuous simulation model results for the pollution control performance of extended detention dry ponds. These results compared very favourably and support the use of analytical models for estimating the pollution control performance of these ponds.

The decision variables in the optimization exercise, as discussed earlier, are the pond depth (h_A), the pond volume (S_A) and the release rate from the pond (Ω). It is evident from Equations 4 and 5 that it is impossible (or extremely difficult, at least) to isolate the variables h_A, S_A and W; therefore, numerical approaches are employed in the optimization. The subsequent sections investigate the behavior of the model and identify characteristics of the behavior that can serve as a guide to employing numerical techniques to solve for optimal pond geometries.

For the purposes of the present work, the following meteorological parameter values were used: λ = 0.282/hr; ψ = 0.023/hr; and ζ = 0.200/mm; these parameters are derived from a statistical analysis of hourly rainfall data for the period 1960 to 1992 from Toronto Pearson International Airport (taken from Adams, 1996). In addition, a commonly used settling velocity distribution of suspended solids in stormwater (MOEE, 1994) was used for the estimation of removal efficiencies in conjunction with a value of 3 for the turbulence factor, n.

OPTIMIZATION OF POND DESIGN IN A SINGLE CATCHMENT

In order to develop a strategy for the optimization of the decision variables (S_A, Ω and h_A) of a stormwater management pond based on a cost function (Equation 2) and a required level of pollution control performance, it is useful to understand the behavior of pollution control with respect to these decision variables.

Since pollution control (C_P) will generally increase for larger pond volumes (Adams, 1996; Papa *et al.*, 1997), and since the cost associated with the implementation of the pond is directly proportional to the pond storage volume (Equation 2), it can be assumed that, for a given pond depth, any pond configuration (i.e., combination of S_A, Ω and h_A) that will achieve a C_P greater than the minimum or target level of pollution control (C_{Pmin}) is a sub-optimal (i.e., more costly) configuration. It therefore follows that the search procedure should be limited to solving combinations of the decision variables which yield pollution control performances at the constraint value, C_{Pmin}.

It is useful to investigate the behavior of pollution control (C_P), for constant pond depths (h_A) and storage volumes (S_A), as a function of the controlled outflow rate (Ω). It should be noted that this exercise is equivalent to varying the average detention time (t_d) of stormwater in the pond. The results of this investigation are illustrated graphically in Figure 2.

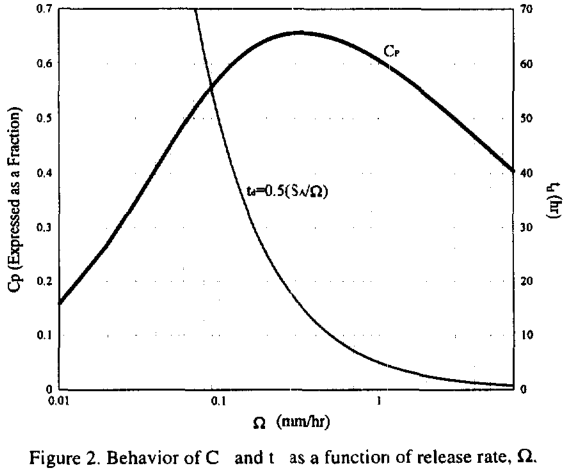

Figure 2. Behavior of C and t as a function of release rate, Ω.

It is evident from Figure 2 that there exists a maximal level of pollution control performance of a pond with a fixed storage volume and depth at a unique rate of outflow and, hence, average detention time. This phenomenon has been previously observed by Papa and Adams (1996). For purposes of this work, it is assumed that there is a minimum level of C_P which must be provided by a pond; therefore, by inspection of Figure 2, it is evident that for a minimum level of pollution control required, say 50% (i.e., $C_P = 0.5$), as well as a given pond volume and depth, there can exist a range of values for W which satisfy the constraint. Furthermore, the fact that there can potentially exist two values of W for a given value of S_A (and h_A) indicates that the pollution control isoquants (plotted in S-Ω space) will be "hooked"; that is, they will contain a minimum value of S_A and hence will not be monotonically decreasing as would be expected from traditional isoquants. Such isoquants are illustrated in Figure 3.

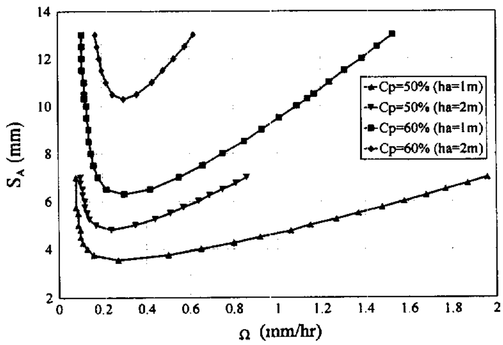

Figure 3. Isoquants of pollution control performance.

It is important for the water resources systems analyst to understand the significance of the shape of the isoquants presented in Figure 3. It is interesting that there are two values of W that provide the same level of long-term pollution control performance for a given storage volume. As the value of W decreases, the detention time of stormwater in the pond increases which, in turn, increases the probability of another runoff event occurring while the pond has not completely drained, thereby increasing the probability of a spill of untreated stormwater into the downstream conveyance system. Conversely, as Ω increases, the probability of untreated spill decreases since the pond drains more quickly and, hence, on a long-term basis, more stormwater is treated albeit at a lower rate. Typically, however, stormwater quality control ponds with longer detention times are favoured.

With this understanding of how pollution control performance behaves for constant pond depths, a strategy can be developed for selecting an optimal combination of storage volume and controlled release rate. Note that this combination will automatically give an optimal average detention time. By visual inspection of the cost function governing a single pond (Equation 2), it is evident that the cost of the pond is directly related to the storage volume (S_A) for a constant pond depth. It therefore follows that in order to minimize cost, the storage volume of the pond must also be minimized. Returning to Figure 3, it is evident that a minimum storage volume exists that can meet the specified pollution control constraint. Therefore, a search procedure to locate this point of minimum storage volume is required as part of the optimization process. This procedure identifies two of the three decision variables required for the design of the pond, namely S_A and Ω.

Next, the pond depth must be incorporated into the optimization framework. Based on the previous discussion, there will exist a unique optimal combination of S_A and Ω for any value of h_A. Also, since the cost of the pond is a function of h_A (see Equation 2), a range of values of h_A must be analyzed to produce the optimal decision variable triplet (h_A, S_A, Ω). In order to develop a search procedure for h_A, it is instructive to investigate the behavior of the cost of the pond as a function of h_A. Figure 4 provides such an illustration. It is important to note that the cost is derived by obtaining an optimal combination of S_A and Ω for each value of h_A analyzed.

The relationship between cost and depth shown in Figure 4 clearly indicates that an optimal pond depth exists. It is conceivable, however, that this optimal pond depth will lie outside a practical design range and thus not be of direct use to the engineer. By examining the cost within pond depth constraints, a minimum cost satisfying the constraints can be obtained. The "shadow price" of the constraint can then be computed if desired by finding the difference in cost between the constrained and unconstrained optimal configurations. Typically, ponds are designed to be deep enough to avoid problems of re-suspension of settled solids. Safety concerns as well as hydraulic considerations may govern the maximum design depth of a pond.

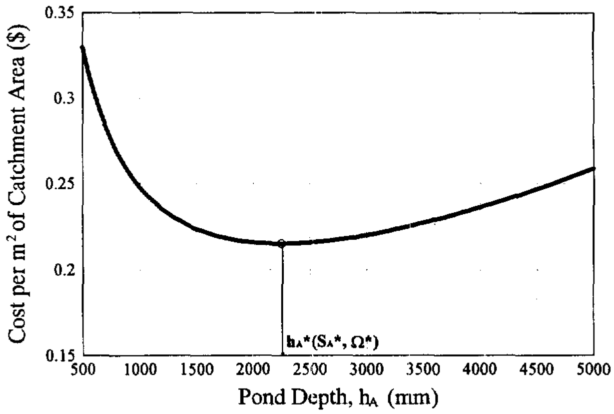

Figure 4. Behavior of pond cost as a function of pond depth.

OPTIMIZATION OF POND DESIGN IN PARALLEL CATCHMENTS

In the previous section, a methodology for estimating the optimal combination of stormwater management pond decision variables (S_A, h_A and Ω) for achieving a specified minimum level of pollution control is developed. That formulation can be further used as an internal component of a larger scale optimization for that of a multiple parallel catchment system as shown in Figure 1. The objective of such an optimization is to minimize the overall cost of all ponds in the system as it may be more economical to provide varying levels of control in the separate catchments to achieve the same level of control in the combined discharge of the system. Meteorological and hydrological characteristics may vary from catchment to catchment as may the costs and other factors such as land availability and public preferences. For the purposes of the present work, the optimization of a three catchment system is undertaken as an illustrative example; however, the methodology may be further generalized to optimize n parallel catchments. Table 1 provides the pertinent data used in the example optimization problem.

The multiple catchment optimization methodology presented herein utilizes a dynamic programming approach. Dynamic programming transforms the multiple catchment (stage) problem into a series of single catchment problems, then combines the solutions of the smaller problems to obtain the solution or policy of the overall (multiple catchment) problem (Mays and Tung, 1992). Although the multiple stage problem is decomposed into single stage problems in dynamic programming, the interdependence among the individual stages is accounted for; that is, the overall pollution control level represented by Equation 1 is not violated by the decomposition.

The objective function of the dynamic programming optimization for the example problem can be expressed as follows:

(6)

subject to the constraint:

$$C_{Ptot} = \frac{\sum_{i=1}^{3}(C_{Pi} \cdot A_{Ci})}{\sum_{i=1}^{3} A_{Ci}} \geq C_{P\,min} \quad \text{(where } C_{Pmin} = 0.5 \text{ or 50\% in the example problem)} \tag{7}$$

Table 1. Data for example problem

	Catchment 1	Catchment 2	Catchment 3
Area (ha)	100	40	70
Runoff Coefficient, ϕ	0.3	0.6	0.7
Cost of Land, P_L ($/m^2)	40	70	90
Construction Cost, P_E ($/m^3)	20	40	60
Minimum Depth, h_A (m)	1.0	1.0	1.0
Maximum Depth, h_A (m)	3.0	3.0	3.0

In this dynamic programming formulation, the stages are the individual catchments tributary to the trunk sewer, the decision variables describe the policy (design) to be implemented at each stage and include the pond depth (h_A), the storage volume of the pond (S_A) and the controlled release rate from the pond (Ω). At each stage of the optimization, there is a state variable which, for this example, is a set of total pollution control (C_{Ptot}) levels which would be provided after that stage.

Based on the amount of pollution control provided prior to the stage under analysis, the level of control required to meet the level specified in the state variable can be determined from Equation 1 after which the optimal pond configuration for that level of control can be determined. In addition, the cost of providing the level of treatment prior to the stage under analysis is added to the least-cost combination solved at the current stage. Also, the target overall level of pollution control provided by the system (C_{Ptot}) is taken to be 0.5 (50%) for this example, where the state variables are incremented by 0.05 (5%) up to the target level.

The dynamic programming forward recursive equation for the example problem is given by

$$f_n^*(S_n) = \underset{d_n}{\mathrm{Min}}\left\{r_n(S_n, d_n) + f_{n-1}^*(S_{n-1})\right\} \tag{8}$$

where S_n is the state variable describing the level of pollution control provided by the system (C_{Ptot}) at stage (catchment) n, d_n is the design (decision variable values) of the pond in catchment n (S_{An}, Ω_n, h_{An}) and r_n is the cost of the pond at stage (catchment) n and $f_n(s_n)$ is the total cost of all ponds in the system at stage n.

At the first stage (catchment 1), the only pollution control that can be provided is by a stormwater management pond in that catchment. That is, in order to provide (say) 35% total pollution control, the pond in catchment 1 must itself provide 73.5% control at an enormous cost of approximately $863,000 due mainly to the extremely large S_A required (i.e., the required S_A in this case would be 14.4 mm which, when converted based on the catchment area is approximately 14,400 m^3). Moreover, it is technically impossible for the pond in catchment 1 alone to provide more than about 47% pollution control for the entire system (since it would be required to provide more than 100% control itself). The spreadsheet application into which the dynamic programming computations were coded automatically substitutes a large cost in such cases to eliminate these alternatives from further consideration. Unfortunately, the details of the dynamic programming computations are too voluminous for presentation herein.

Similarly, in order to provide 25% overall pollution control after stage 2 and after having provided 10% overall pollution control at stage 1 (equivalent to 21% C_P in catchment 1 alone), the pond in catchment 2 must itself provide 78.75% pollution control. The final results of the optimization presented in this section are summarized in Table 2 which gives the optimal design policy to be followed in an attempt to minimize costs while meeting the water quality objectives downstream; the minimum cost determined using the dynamic programming optimization procedure is approximately $653,000. To evaluate the results of the optimization, it should be noted that if 50% control were provided in each of the catchments the total cost would be approximately $779,000 (i.e., 19% more costly than that obtained by the optimization procedure).

Table 2. Optimal design policy

	Catchment 1	Catchment 2	Catchment 3
C_P* (%)	63.0	52.5	30.0
h_A* (m)	1.00	1.70	3.00
S_A* (mm)	4.74	6.18	2.68
Ω* (mm/hr)	0.19	0.30	0.22

RECOMMENDATIONS AND CONCLUSIONS

Optimization techniques for the planning and design of urban drainage systems which seek to minimize cost can be of practical importance to land developers and municipalities alike. Land developers are typically confronted with environmental regulations which must be met, and the implementation of stormwater management ponds can significantly impact the land available for development; hence, an optimization procedure can be of direct benefit to the developer. Municipalities often take the lead in the development of master servicing plans at a larger scale which typically encompass multiple catchments and the planning of district sewerage and storage facilities; therefore, an optimization procedure which incorporates several catchments is required to minimize the capital expenditure and relieve the burden on the tax-payer.

This paper provides a framework for the optimization of a single catchment, single pond system using computationally efficient analytical probabilistic models for stormwater management planning and design which can be incorporated into a dynamic programming formulation for the optimization of a multiple catchment, multiple pond system.

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